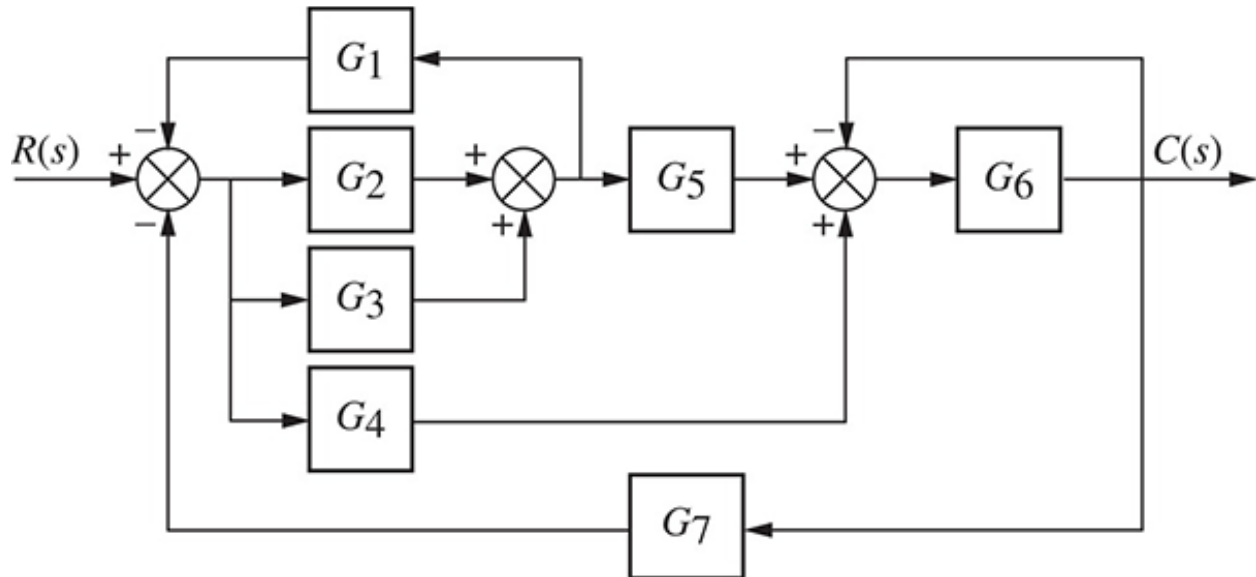


### Homework 3

#### Question 1

Reduce the block diagram shown below to a single transfer function,  $T(s) = C(s)/R(s)$ .



## Question 2

A common application of dc motors is in accurate positioning of a mechanical load. A schematic diagram of a possible arrangement is shown in Figure below.

- Draw a suitable linear graph for the entire system shown in Figure below, mark the variables and parameters (you may introduce new, auxiliary variables but not new parameters), and orient the graph.
- Give the number of branches ( $b$ ), nodes ( $n$ ), and the independent loops ( $l$ ) in the complete linear graph. What relationship do these three parameters satisfy? How many independent node equations, loop equations, and constitutive equations can be written for the system? Verify the sufficiency of these equations to solve the problem.
- Take current through the inductor ( $i_L$ ), speed of rotation of the motor rotor ( $\omega_r$ ), torque transmitted through the load shaft ( $T_k$ ), and speed of rotation of the load ( $\omega_l$ ) as the four state variables; the armature supply voltage  $v_a(t)$  as the input variable; and the shaft torque  $T_k$  and the load speed  $\omega_l$  as the output variables. Write the independent node equations, independent loop equations, and the constitutive equations for the complete linear graph. Clearly show the state-space shell.
- Eliminate the auxiliary variables and obtain a complete state-space model for the system, using the equations written in Part (c) above. Express the matrices  $A$ ,  $B$ ,  $C$ , and  $D$  of the state space model only in terms of the system parameters  $R_a$ ,  $L_a$ ,  $k_m$ ,  $J_r$ ,  $k_t$ ,  $b_l$ , and  $J_l$ .

